Midterm 2 - Review - Answers

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1) -44

2) 0 (row-reduce and notice that at some point you get 2 identical rows, hence A is not invertible, hence it has determinant 0)

3)

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & -2 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Any other order/multiple of the columns P is fine too, as long as you remember that every eigenvector has to go with the corresponding eigenvalue!

4) **NO** (doesn't contain the **0**-vector)

5) **YES** (span of anything is a vector space)

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$$

$dim(V) = 3$

6) (a) Basis for $Row(A)$:

$$\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ -3 \\ 6 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 3 \end{bmatrix} \right\}$$

Basis for $Col(A)$:

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 5 \\ -4 \end{bmatrix} \right\}$$

(b) $Rank(A) = 3$ (number of pivots), $Dim(Nul(A)) = 5 - Rank(A) = 2$

7) (a) Show $T(p+q) = T(p) + T(q)$ and $T(cp) = cT(p)$

(b)

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & \\ 1 & 0 & 0 & 0 & \\ 0 & \frac{1}{2} & 0 & 0 & \\ 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \end{bmatrix}$$

8)

$$\mathcal{C} \overset{P}{\leftarrow} \mathcal{B} = \begin{bmatrix} -3 & 2 \\ -4 & 3 \end{bmatrix}, [\mathbf{x}]_{\mathcal{C}} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

9) $\hat{\mathbf{y}} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

10)

$$\mathcal{B} = \left\{ \frac{1}{\sqrt{12}} \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{12}} \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{12}} \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$$

11) $\hat{x} = \begin{bmatrix} 3 \\ \frac{1}{2} \end{bmatrix}$, Error: $\sqrt{2}$

12) (a) **FALSE** (for example, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$)

(b) **FALSE** ($Q^TQ = I$, but not necessarily $QQ^T = I$, for example, take $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$)

(c) **FALSE** (it's the opposite!)

(d) **FALSE** (for example, take the $x-$ axis and the $y-$ axis in $\mathbb{R}^2$)

(e) **TRUE** (see $HW5$)

(f) **TRUE** (suppose $A\mathbf{v} = \lambda\mathbf{v}$ for some $\mathbf{v} \neq \mathbf{0}$, calculate $A^2\mathbf{v}$ in two different ways)

(g) **FALSE** (take $A =$ zero-matrix)

(h) **TRUE** (basis theorem)

(i) **TRUE** (A similar to B means $A = PBP^{-1}$ for some invertible P)

(j) **TRUE** (use the previous question)

(k) **TRUE** (draw a picture)